* S B - 2 7 3 5 / 1 0 0 *

SB-2735

M. Sc. - I (Sem. II) Examination

March / April – 2011

Mathematics

(Discrete Structure)

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.	Seat No. :
Fillup strictly the details of ☛ signs on your answer book.	
Name of the Examination :	
☛ **M. Sc. - I (Sem. II)**	
Name of the Subject :	
☛ **Mathematics**	Student's Signature
☛ Subject Code No. : **2 7 3 5** ☛ Section No. (1, 2,.....) : **Nil**	

(2) Attempt all question.

(3) Follow the usual notations and conventions.

(4) Figures on the right indicate full marks.

1 Attempt any **two** : **14**

(1) Let $\langle M,*\rangle$ be a monoid. Prove that there enists a subset $T\subseteq M^M$ such that $\langle M,*\rangle$ is isomorphic to the monoid $\langle T,O\rangle$ where M^M be the set of all monoid automorphisms.

(2) Let $\langle L,\leq\rangle$ be a complemented distributive lattice. Prove that $a\leq b\Leftrightarrow a*b^1=0\Leftrightarrow a^1\oplus b=1\Leftrightarrow b^1\leq a^1$ where $a,b\in L$.

(3) In a distributive lattice $\langle L.*.\oplus\rangle$, show that

(a) $(a*b=a*c)\wedge(a\oplus b=a\oplus c)\Rightarrow b=c$

(b) $(a*b)\oplus(b*c)\oplus(c*a)=(a\oplus b)*(b\oplus c)*(c\oplus a)$

where $a,b,c\in L$.

2 Attempt any **three** : 14

(1) In a monoid, show that the set of all left invertible form a submonoid.

(2) Show that in a bounded distributive lattice, the element which have complements form a sublattice.

(3) Let Sn be the set of divisors of n and D be the relation of division. Prove that $\langle S_{30}, D \rangle$ is a lattice and draw Hasse-diagram of it.

(4) Define lattice homomorphism, order-preserving and order isomorphic.

3 Attempt any **three** : 14

(1) Use the karnaugh map representation to find a minimal sum of product expression of f(a,b,c,d) = $\sum(0,5,7,8,12,14)$.

(2) Show that the following expressions are equivalent and obtain their sum of products canonical form

(a) $(x \oplus y) * (x^1 \oplus z) * (y \oplus z)$

(b) $(x * z) \oplus (x^1 * y)$.

(3) What is grammer ? Write grammer for the language "the set of non-negative odd integers."

(4) Let $\langle B, *, \oplus, ', o, 1 \rangle$ and $\langle P, \cap, \cup, \sim, \alpha, \beta \rangle$ be two Boolean algebras. If a mapping $f : B \to P$ preserves the operations * and ' then prove that it is a Boolean homomorphism.

4 Attempt any **three** : 14

(1) In any Boolean algebra, prove that $a = b \Leftrightarrow ab^1 + a^1 b = 0$.

(2) Let S be any non-empty set and f(s) be its power set. Prove that $\langle f(s), \cap, \cup, \sim, \varnothing, s \rangle$ is a Boolean algebra in which the complement of $A \subset S$ is ~A=S-A.

(3) Use the Quine-McCluskey algorithm to find the minimal expression of $f(a,b,c,d) = \sum(o, 5, 10, 13, 15)$.

(4) Prove that the rank of any well-formed polish formula is 1 and the rank of any proper head of a polish formula is greater than of equal to 1.

5 Attempt any **three**. **14**

(1) Simplify the following Boolean expressions

(a) $(a*b)^1 \oplus (a \oplus b)^1$

(b) $(a^1 * b^1 * c^1) \oplus (a * b^1 \not{v} c) \oplus (a * b^1 * c^1)$.

(2) For $f(x, y, z, w) = w^1 + y(x^1 + z^1)$ represent in the circuit diagram, cube and truth table.

(3) Convert the infix notation a/b*c to reverse polish notation.

(4) Shor that the lattice $\langle S_n, D \rangle$ for n=216 is isomorphic to the direct product of two lattices for n=8 and n=27.
